Sydney Technical High School

Student Name: ______________________________

Teacher's Name: ______________________________

Year 12
Mathematics Advanced

Trial Examination

August 2023

General Instructions

- Working time –3 hours + 10 minutes reading time
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- In questions **11-31**, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless setting out or illegible writing

Total marks: 90

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11-31
- Allow about 2 hours and 45 minutes for this section

For marker's use only:

MC	11-16	17 – 19	20 – 23	24-26	27 – 29	30 – 31
/10	/16	/17	/15	/15	/14	/13

Total	/100	%

Section I – Multiple Choice

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Each multiple choice question is worth 1 mark.
Clearly colour in each bubble of the correct answer on your multiple choice sheet in your answer booklet.

Question One

What is the gradient of the line with equation $3x - 7y + 1 = 0$

A) $-\frac{7}{3}$

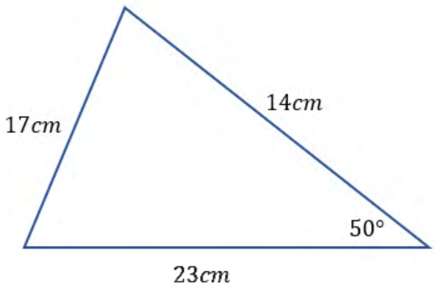
B) $-\frac{3}{7}$

C) $\frac{3}{7}$

D) $\frac{7}{3}$

Question Two

What is the area of the following triangle:



A) $Area = \frac{1}{2} \times 14 \times 17 \times \cos 50$

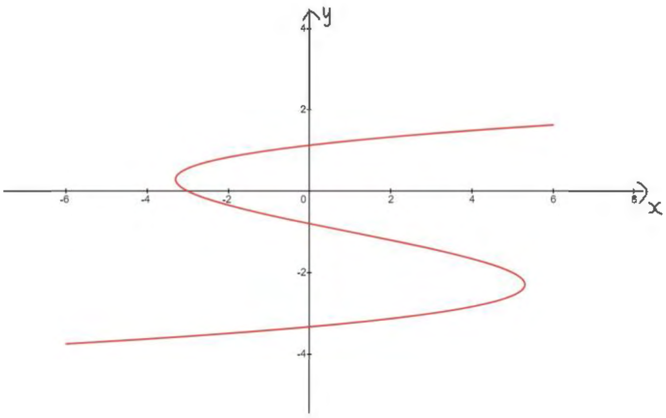
B) $Area = \frac{1}{2} \times 14 \times 23 \times \cos 50$

C) $Area = \frac{1}{2} \times 14 \times 17 \times \sin 50$

D) $Area = \frac{1}{2} \times 14 \times 23 \times \sin 50$

Question Three

Which type of relationship best describes the curve:



A) One-to-one

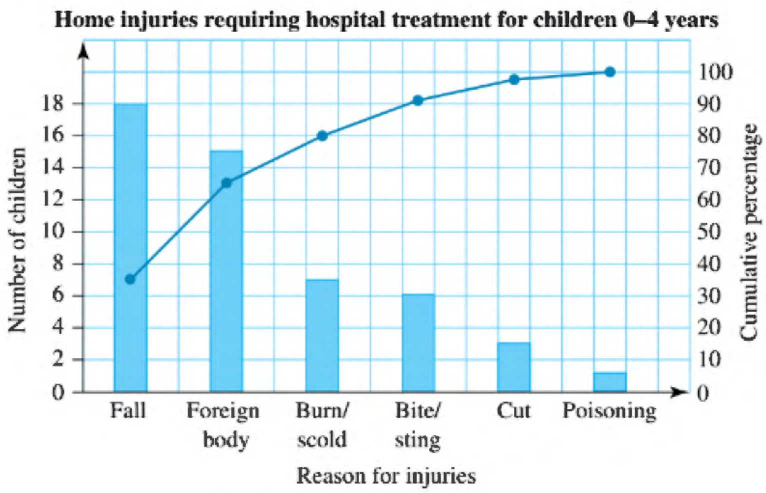
B) One-to-many

C) Many-to-one

D) Many-to-many

Question Four

The Pareto Chart below shows the causes of home injuries for which children aged 0 to 4 years required hospital treatment in March 2023.
What percentage of children required hospital treatment in March 2023 for burn/scald injuries?



A) 7%

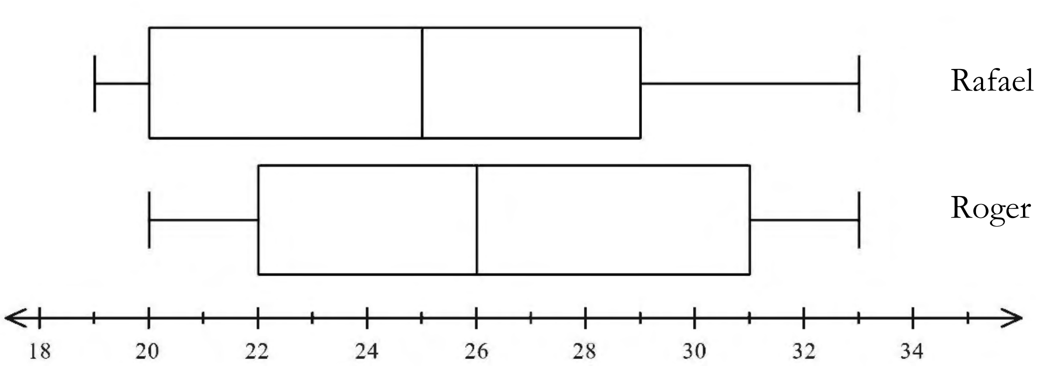
B) 14%

C) 40%

D) 80%

Question Five

The parallel box plots below are used to compare the time taken to complete a set of tennis (to the nearest minute), by two players over their last 40 games.



Which is a true statement about the two sets of data?

A) They have the same interquartile range.

B) They have the same median.

C) They have the same range.

D) They have the same upper quartile.

Question Six

The graph of a function $y = \frac{4}{x+1}$ is translated 6 units right, and dilated vertically by a factor of $\frac{1}{3}$.

Which of the following gives the equation of the new function?

A) $\frac{y}{3} = \frac{4}{x-5}$

B) $3y = \frac{4}{x-6}$

C) $3y = \frac{4}{x-5}$

D) $\frac{y}{3} = \frac{4}{x-6}$

Question Seven

Three chess players are chosen from 4 males and 6 females.
What is the probability that they are all the same gender?

A) $\frac{1}{5}$

B) $\frac{3}{10}$

C) $\frac{2}{5}$

D) $\frac{1}{2}$

Question Eight

What is the domain of the function:

$$y = \sqrt{1 - 3x}$$

A) $\left(-\infty, \frac{1}{3}\right)$

B) $\left(\frac{1}{3}, \infty\right)$

C) $\left(-\infty, \frac{1}{3}\right]$

D) $\left[-\infty, \frac{1}{3}\right]$

Question Nine

What is the solution of $9^x = 2$

A) $x = \frac{\log_e 2}{9}$

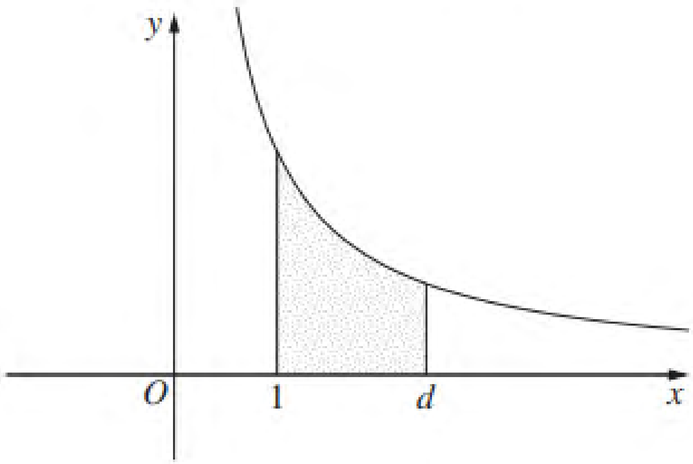
B) $x = \frac{2}{\log_e 9}$

C) $x = \log_e(\frac{2}{9})$

D) $x = \frac{\log_e 2}{\log_e 9}$

Question Ten

The diagram shows the area under the curve $y = \frac{2}{x}$ from $x = 1$, to $x = d$.



What value of d makes the shaded area equal to 2?

A) e

B) $e + 1$

C) $2e$

D) e^2

Section II – Short Answer

90 marks

Attempt Question 11-31

Allow about 2 hours and 45 minutes for this section

In Section II, your responses should include all relevant mathematical reasoning and/or calculations.

Question Eleven

Find values for $\cos\theta$ if $\sin\theta = \frac{2}{\sqrt{29}}$ and θ is obtuse. 2

Question Twelve

Find $\int 3 + \sin 4\pi x \ dx$ 2

Question Thirteen

Jamie owns a company producing and selling backpacks. The income function of $y = 80x$ is shown on the graph below, where x is the number of backpacks sold. The cost of producing these backpacks includes a set up cost of \$4 500 and additional costs of \$30 per backpack.

a) Write the cost function in the form $y = mx + c$ 1

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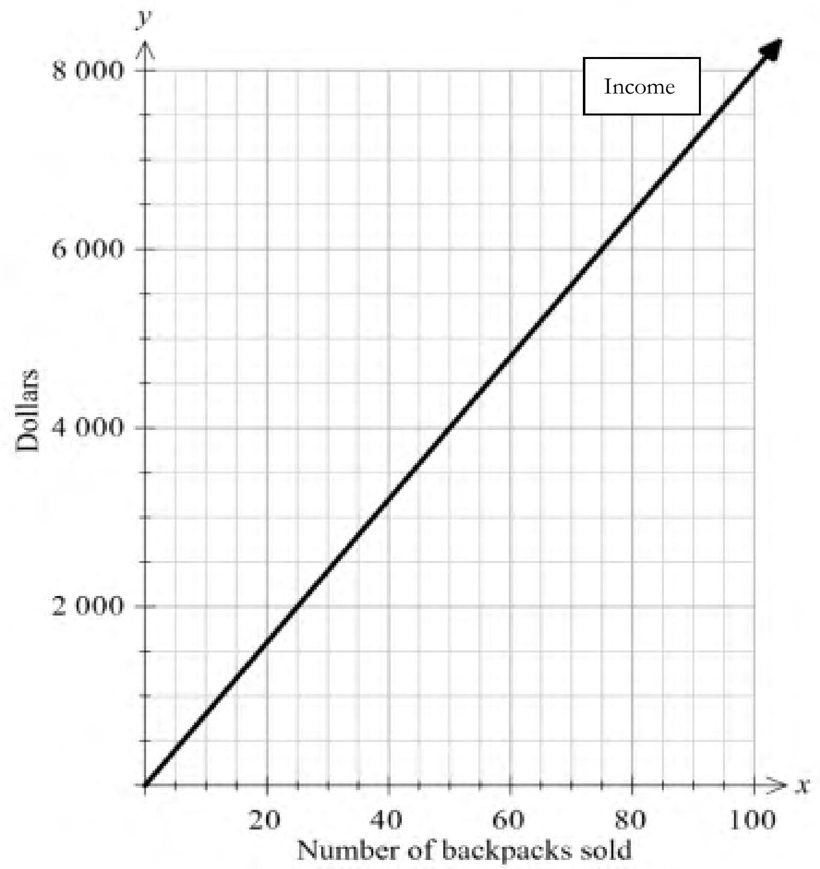
b) Sketch the cost function on the axis below: 2



c) Hence, determine Jamie's break even point. 1

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Question Fourteen

A polynomial function $f(x) = x^3 - 5x^2 + 8x - 3$ is defined over the domain $0 \le x \le 4$.

a) What is the degree of this polynomial? 1

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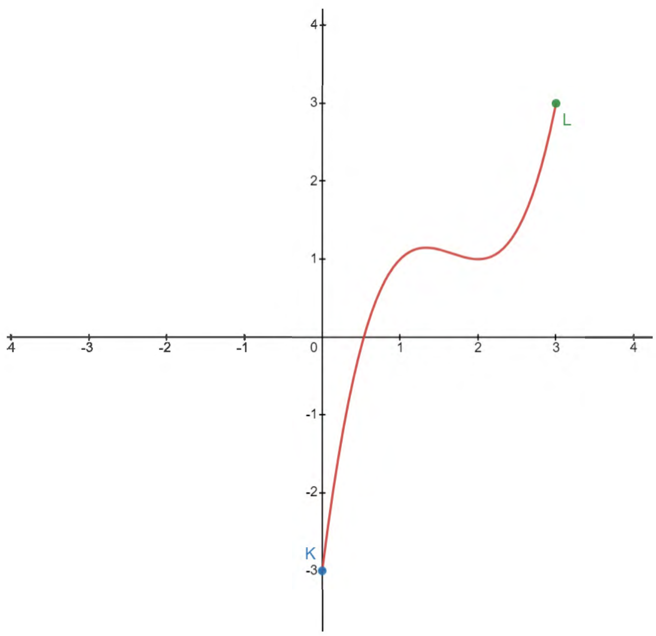
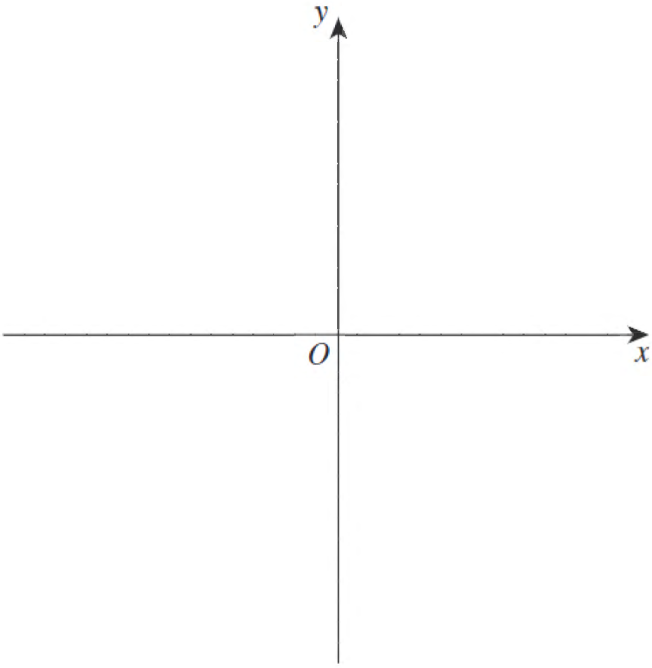
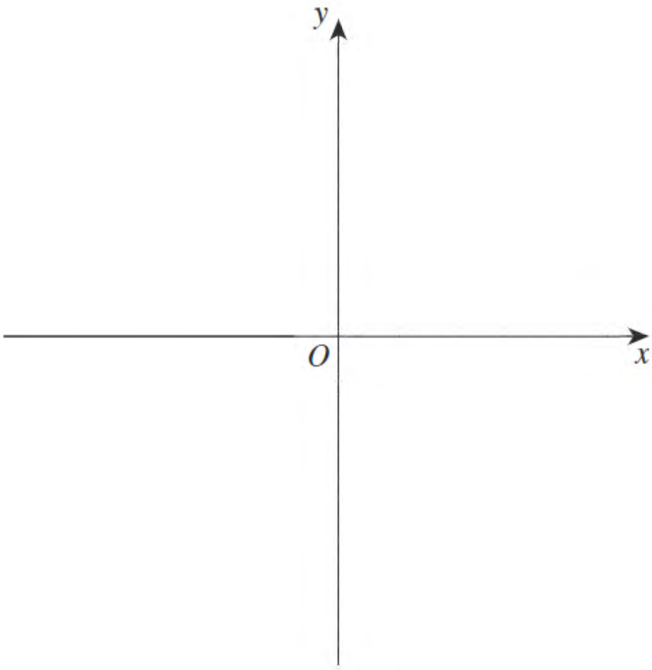
b) $y = f(x)$ is sketched below. $K = (0, -3)$ and $L = (3, 3)$



i. Sketch the graph of $y = -f(x)$ 1



ii. Sketch the graph of $y = f(-x)$ 1



Question Fifteen

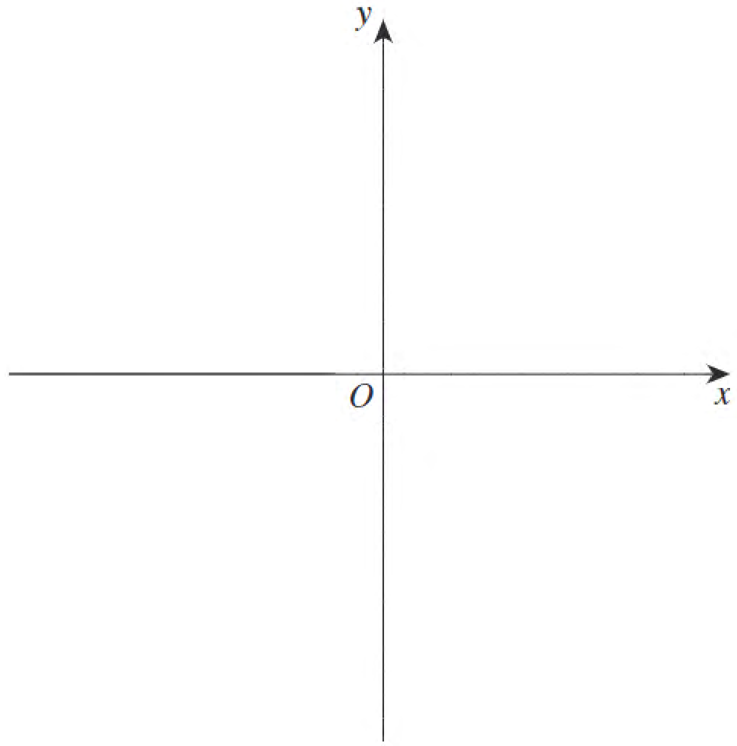
Prove that $sin^4\theta - cos^4\theta = 2sin^2\theta - 1$ 3

Question Sixteen

The function $y = f(x)$ is continuous for all values of x. The following is known of the functions' properties. 2

- When $x < 2$, $f'(x) < 0$ and $f''(x) > 0$
- When $x > 2$, $f'(x) < 0$ and $f''(x) < 0$
- $f(2) = f'(2) = f''(2) = 0$

Sketch the function on the axis below.



Question Seventeen

Solve $\ln(7x - 12) = 2\ln x$. 2

Question Eighteen

Consider the quadratic function:

$$y = x^2 - x + 4$$

a) How many real roots does the function have? 2

b) What are the coordinates of the vertex? 2

c) Sketch the graph of the parabola. 2

Question Nineteen

Differentiate:

a) $y = \frac{4e}{3x^2}$ 2

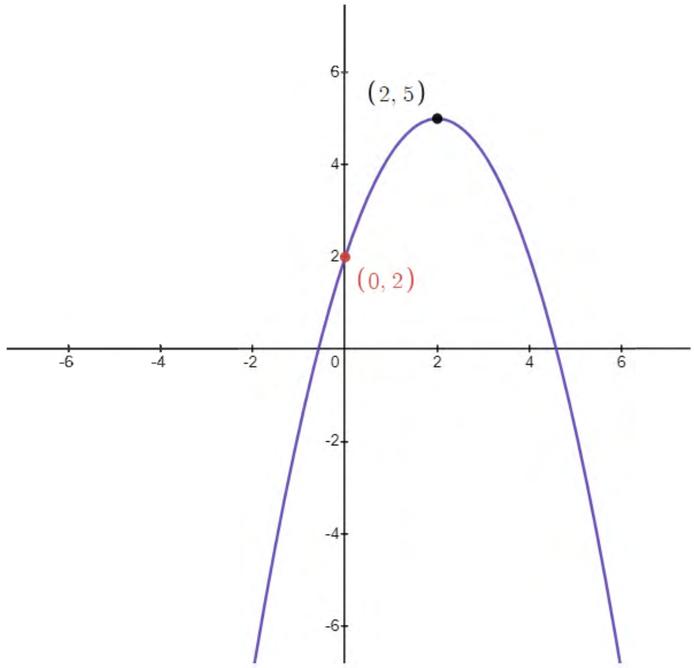
b) $f(x) = x^2 \log_e 2x$ 2

c) $y = \frac{\ln x}{x}$ 2

d) $y = cos^3(x^2 + 1)$ 3

Question Twenty

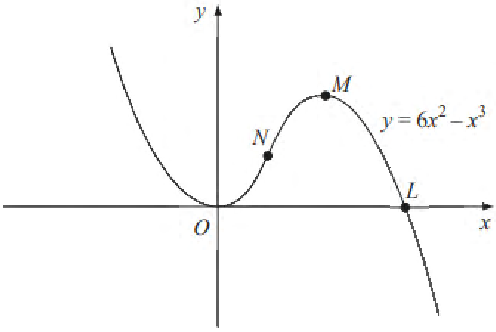
The function $f(x) = x^2$ is transformed into a new function whose graph is shown below: 2



Find the equation of the new function in the form $g(x) = k(x + b)^2 + c$ for some constants $k, b,$ and c.

Question Twenty-One

The diagram shows a sketch of the curve $y = 6x^2 - x^3$. The curve cuts the x axis at L, has a local maximum at M, and a point of inflexion at N.



a) Find the coordinates of L. 2

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b) Find the coordinates of M. 3

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c) Find the coordinates of N. 2

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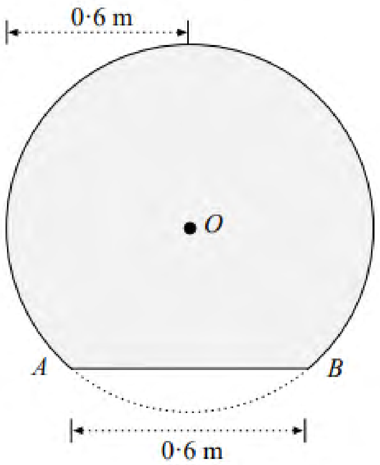
Question Twenty-Two

A table-top is in the shape of a circle with a small segment removed as shown. 3
The circle has centre O and radius 0.6 metres. The length of the straight edge AB is also 0.6 metres.
Find the area of the table-top, leaving your answer in exact form.



NOT TO SCALE

Question Twenty-Three

A small group of people were surveyed to determine whether they were part of the local basketball or curling team. 3

- 6 people played basketball only.
- 13 people played curling only.
- 3 people played neither.
- k people played both.

Let B be the event that a person plays basketball and let C be the event that a person plays curling.
Determine the value of k such that the events B and C are independent.

[**Curling**: a game played on ice, especially in Scotland and Canada, in which large round flat stones are slid across the surface towards a mark. Members of a team use brooms to sweep the surface of the ice in the path of the stone to control its speed and direction.]

Question Twenty-Four

A particle moves on a straight line. When it is x metres from the origin, its velocity is given by $v = 4 - e^{\frac{-t}{2}}$, the time t being measured in seconds.

a) Find its acceleration at time t. 1

b) Initially the particle was at the origin. Find its displacement from the origin after 4 seconds. Leave your answer in metres, correct to 2 decimal places. 3

Question Twenty-Five

Sketch the graph of $y = x^3 - 6x^2 + 9x - 5$ in the space provided showing all: 5

- turning points
- points of inflexion
- y-intercept

There is no need to find the x-intercept(s).

Question Twenty-Six

A probability density function f for a random variable X is given by:

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{9+2x}}\,, & 0 \le x \le 8 \\ 0\,, & \text{otherwise} \end{cases}$$

where k is a constant.

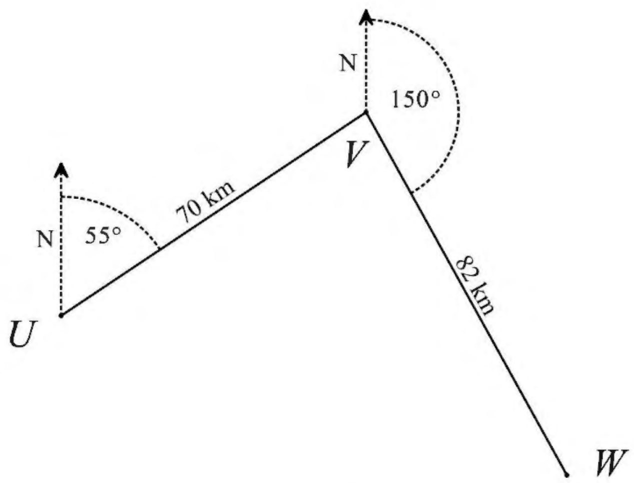
a) Show that $k = \frac{1}{2}$. 2

b) Find the cumulative distribution function $F(x)$. 2

c) Find the median. 2

Question Twenty-Seven

A helicopter leaves Underwood and flies 70 km on a bearing of $055°$ to Vanna Beach.
It then flies 82 km on a bearing of $150°$ to Weston.
The diagram below illustrates the journey.

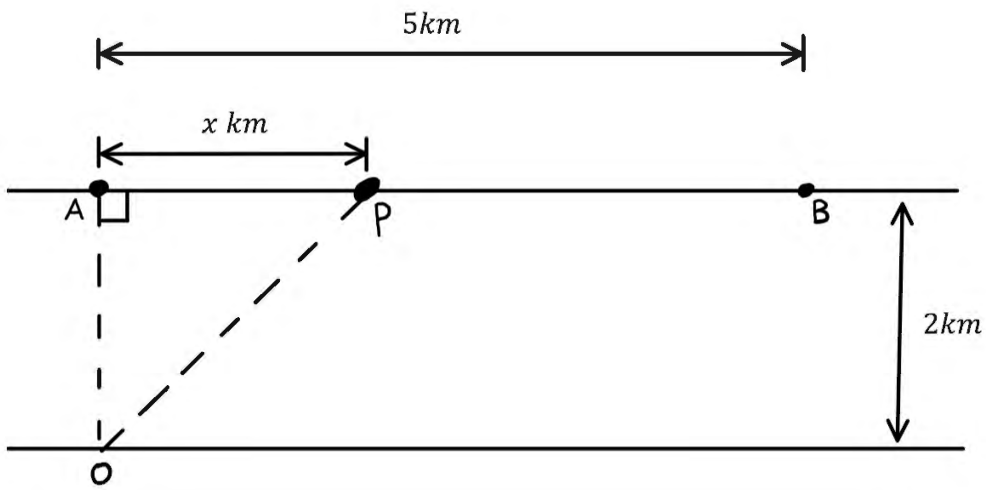


a) It then plans to fly directly back to Underwood. Calculate, to the nearest km, the distance that it will fly. 2

b) Determine the bearing from Weston on which it should fly to return to Underwood. Give your answer to the nearest degree. 2

Question Twenty-Eight

The diagram shows a straight section of a river, two kilometres wide. Gonzo is at a point O on one bank and he wishes to reach a point B on the opposite bank. The point A is directly opposite O and the distance from A to B is 5 kilometres.
Gonzo can row at a speed of 8 km/h, and jog at 12 km/h. He intends to row in a straight line to a point P on the opposite bank and then jog directly from P to B.
Let the distance AP be x km.



a) Show that the time T, in hours, that Gonzo takes to reach B is given by $T = \frac{\sqrt{x^2+4}}{8} + \frac{5-x}{12}$. 2

b) Find the distance AP for which Gonzo's journey is minimised. 4

Question Twenty-Nine

Data was collected from 6 people detailing the number of black cats they have seen per year and the average number of car accidents they are involved in per year.

Number of black cats seen in a year (x)	4	6	8	13	17	20
Number of car accidents per year (y)	1	4	2	5	7	9

a) Calculate Pearson's correlation coefficient correct to 2 d.p. 1

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b) Write the equation of the least squares regression line, writing your values to 2 d.p. 1

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c) If a person was in 6 car accidents in a year, how many black cats would you expect them to have seen. Answer to the nearest cat. 1

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d) Antonia looked at this data and made the statement "Black cats are unlucky, therefore the more that a person has seen, the more car accidents they have been in." 1
Based on the data and your knowledge of statistics, justify whether you agree or disagree with this statement.

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Question Thirty

Data collected for a Speedrun of the video game Legend of Zelda: Ocarina of Time resulted in a mean of 4 hours 40 minutes 20 seconds, with a standard deviation of 12 minutes 27 seconds. Elliot saw that there was a competition with prizes on offer for any entry that can complete the Speedrun faster than 93% of people. 3

An extract from a probability table for the standard normal distribution is shown below.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

Elliot completed the Speedrun in 4 hours 21 minutes and 47 seconds.
Did he qualify for a prize?
Justify your answer through your knowledge of the normal distribution and z-scores.

Question Thirty-One

The planet Tatooine has Twin Suns that it orbits around.
The rising and setting of these suns can be modelled with the functions:

Sun 1	$f(x) = -2\cos\frac{x}{4}$
Sun 2	$g(x) = 2\sin(\frac{x}{4} - \frac{5\pi}{8})$

a) Neatly sketch the two equations in the space provided, between $0 \leq x \leq 8\pi$. **3**
Note: The two graphs intersect when $x = \frac{\pi}{4}$ and when $x = \frac{17\pi}{4}$ in the given domain. DO NOT PROVE THIS.

b) The horizon is represented by the x-axis. 3
Work cannot be done on the planet when either of the two suns are in the top half of their path across the sky. For how many hours must work stop each day?

c) If the area under the curves bounded by the x-axis represents the total surface area of Tatooine experiencing sunlight (in millions of km^2), find the area of the planet that receives daylight for the given cycle. Give your answer to 2 d.p. 4

-End of Examination-

Sydney Technical High School

MULTIPLE CHOICE

ANSWER SHEET

Completely fill the response oval representing the most correct answer.

Do not remove this sheet from the answer booklet.

Mathematics Advanced 2023 Trial Solutions

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ○	D ●
3.	A ○	B ●	C ○	D ○
4.	A ○	B ●	C ○	D ○
5.	A ●	B ○	C ○	D ○
6.	A ○	B ○	C ●	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ●	B ○	C ○	D ○

Section II – Short Answer

90 marks

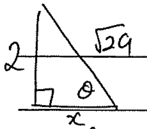
Attempt Question 11-31

Allow about 2 hours and 45 minutes for this section

In Section II, your responses should include all relevant mathematical reasoning and/or calculations.

Question Eleven

Find values for $\cos\theta$ if $\sin\theta = \frac{2}{\sqrt{29}}$ and θ is obtuse. 2



$x^2 = (\sqrt{29})^2 - 2^2$

$x = \sqrt{25}$

$x = 5$

$\therefore \cos\theta = \frac{-5}{\sqrt{29}}$

Question Twelve

Find $\int 3 + \sin 4\pi x \; dx$ 2

$\int 3 + \sin 4\pi x \, dx = 3x - \frac{\cos 4\pi x}{4\pi} + C$

* Mark for +C

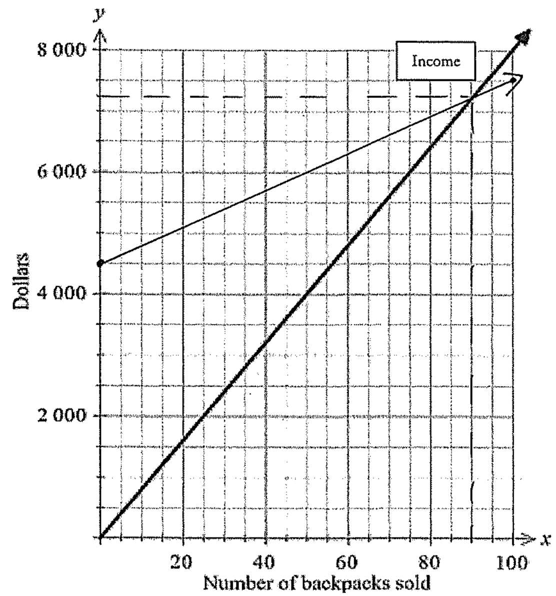
Question Thirteen

Jamie owns a company producing and selling backpacks. The income function of $y = 80x$ is shown on the graph below, where x is the number of backpacks sold. The cost of producing these backpacks includes a set up cost of $4 500 and additional costs of $30 per backpack.

a) Write the cost function in the form $y = mx + c$ 1

$y = 30x + 4500$

b) Sketch the cost function on the axis below: 2



when $x = 100$

$y = 30(100) + 4500$

$y = 7500$

c) Hence, determine Jamie's break even point. 1

From graph only!

Break even when $x = 90$

$y = 7200$

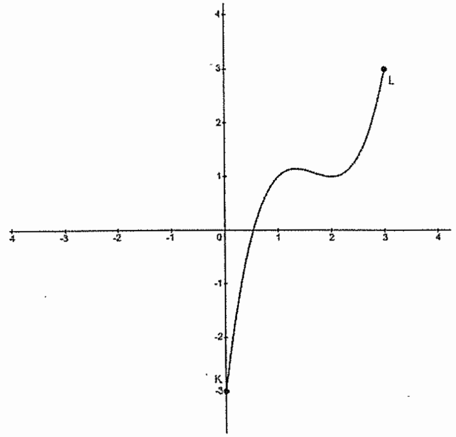
Question Fourteen

A polynomial function $f(x) = x^3 - 5x^2 + 8x - 3$ is defined over the domain $0 \leq x \leq 4$.

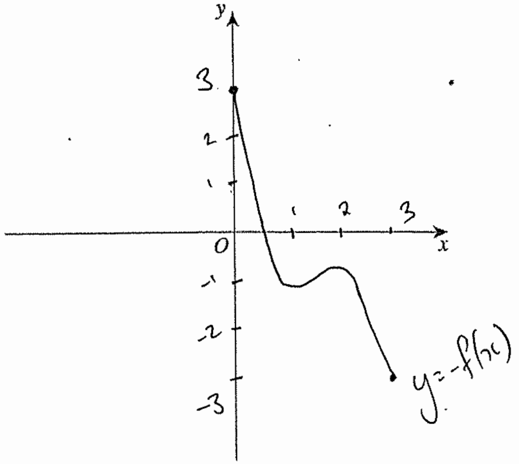
a) What is the degree of this polynomial? 1

3

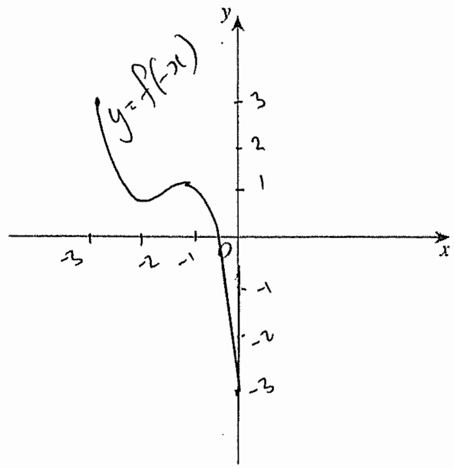
b) $y = f(x)$ is sketched below. $K = (0, -3)$ and $L = (3, 3)$



i. Sketch the graph of $y = -f(x)$ 1



ii. Sketch the graph of $y = f(-x)$ 1



Question Fifteen

Prove that $sin^4\theta - cos^4\theta = 2sin^2\theta - 1$ 3

$LHS = (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= (\sin^2\theta - (1 - \sin^2\theta)) \times 1$

$= 2\sin^2\theta - 1$

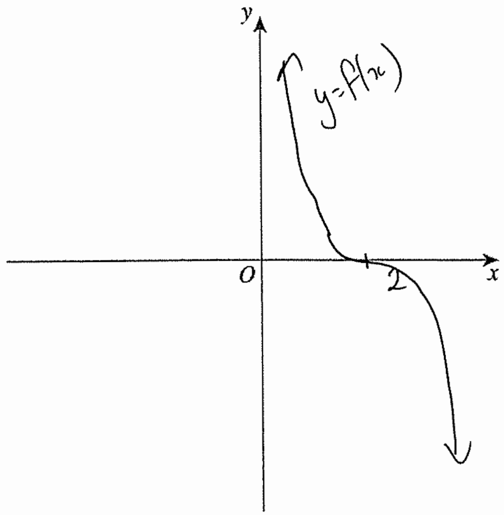
$= RHS$

Question Sixteen

The function $y = f(x)$ is continuous for all values of x. The following is known of the functions' properties. 2

- When $x < 2, f'(x) < 0$ and $f''(x) > 0$ decreasing, concave up
- When $x > 2, f'(x) < 0$ and $f''(x) < 0$ decreasing, concave down
- $f(2) = f'(2) = f''(2) = 0$

Sketch the function on the axis below.



Question Seventeen

Solve $\ln(7x - 12) = 2\ln x$. 2

$\ln(7x-12) = \ln(x^2)$

$7x - 12 = x^2$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$\therefore x = 3, x = 4$

$7(3) - 12 = 9 > 0 \therefore$ True

$7(4) - 12 = 16 > 0 \therefore$ True

Question Eighteen

Consider the quadratic function:

$$y = x^2 - x + 4$$

a) How many real roots does the function have? 2

$\Delta = b^2 - 4ac$

$\Delta = (-1)^2 - 4(1)(4)$

$= 1 - 16$

$= -15 < 0$ $\therefore$ no real roots.

b) What are the coordinates of the vertex? 2

axis $= \frac{-b}{2a}$

$= \frac{1}{2}$

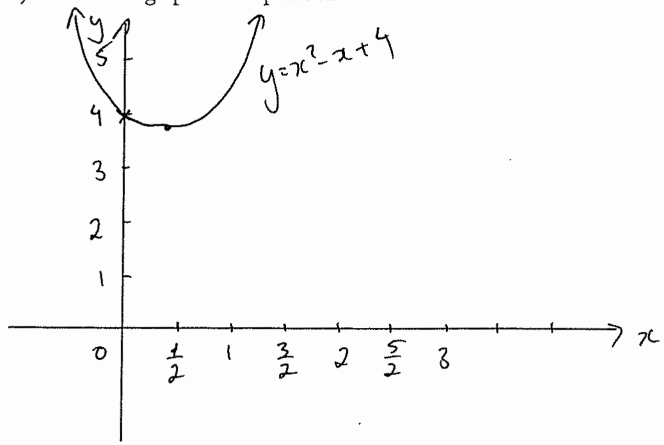
when $x = \frac{1}{2}$

Vertex: $y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 4$

$y = \frac{15}{4}$ (or $3\frac{3}{4}$)

$\therefore$ Vertex $\left(\frac{1}{2}, \frac{15}{4}\right)$

c) Sketch the graph of the parabola. 2



Question Nineteen

Differentiate:

a) $y = \frac{4e}{3x^2}$ 2

$y = \frac{4e}{3}x^{-2}$

$\frac{dy}{dx} = \frac{-8e}{3}x^{-3}$

$= \frac{-8e}{3x^3}$

b) $f(x) = x^2 \log_e 2x$ 2

$f'(x) = x^2 \cdot \frac{1}{x} + 2x \cdot \log_e 2x$ $\quad u = x^2 \quad v = \log_e 2x$

$u' = 2x \quad v' = \frac{2}{2x} = \frac{1}{x}$

$= x + 2x \log_e 2x$

c) $y = \frac{\ln x}{x}$ 2

$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$ $\quad u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$

$= \frac{1 - \ln x}{x^2}$

d) $y = \cos^3(x^2 + 1)$ 3

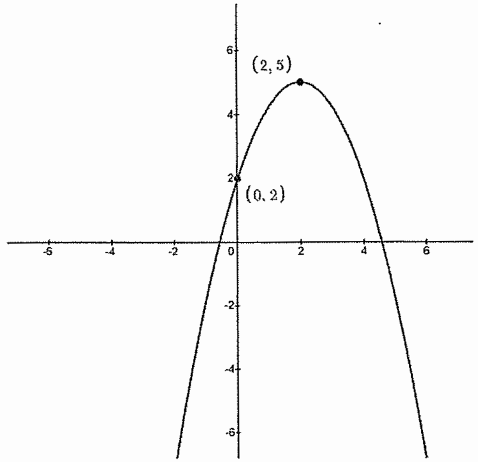
$y = [\cos(x^2+1)]^3$

$\frac{dy}{dx} = -3 \cdot 2x \sin(x^2+1) \cdot [\cos(x^2+1)]^2$

$= -6x \sin(x^2+1)\cos^2(x^2+1)$

Question Twenty

The function $f(x) = x^2$ is transformed into a new function whose graph is shown below: 2



Find the equation of the new function in the form $g(x) = k(x + b)^2 + c$ for some constants k, b, and c.

Vertex $f(x) = x^2 \rightarrow (0,0)$

shift to $\rightarrow (2,5)$

$g(x) = k(x-2)^2 + 5$

when $x = 0 \quad y = 2$

$2 = k(0-2)^2 + 5$

$2 = 4k + 5$

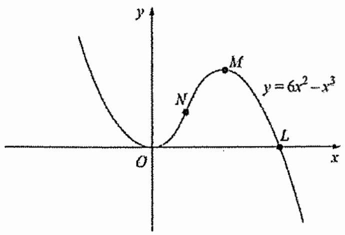
$4k = -3$

$k = -\frac{3}{4}$

$\therefore g(x) = -\frac{3}{4}(x-2)^2 + 5$

Question Twenty-One

The diagram shows a sketch of the curve $y = 6x^2 - x^3$. The curve cuts the x axis at L, has a local maximum at M, and a point of inflexion at N.



a) Find the coordinates of L. 2

$y = 6x^2 - x^3$
when $y = 0$
$0 = 6x^2 - x^3$
$x^2(6-x) = 0$
$\therefore x = 0, \; x = 6$
$\therefore L(6, 0)$

b) Find the coordinates of M. 3

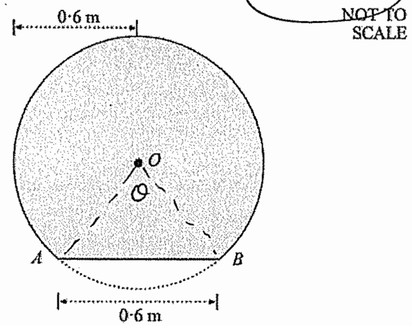
$\frac{dy}{dx} = 12x - 3x^2$
when $\frac{dy}{dx} = 0$
$12x - 3x^2 = 0$
$3x(4 - x) = 0$
$\therefore x = 0, \; x = 4$
M not at origin $\therefore M \rightarrow x = 4$
$y = 6(4)^2 - 4^3$
$= 32$
$\therefore M(4, 32)$

c) Find the coordinates of N. 2

$\frac{d^2y}{dx^2} = 12 - 6x$
when $\frac{d^2y}{dx^2} = 0$
$0 = 12 - 6x$
$6x = 12$
$x = 2$
when $x = 2$
$y = 6(2)^2 - 2^3$
$= 16$
$\therefore N(2, 16)$

Question Twenty-Two

A table-top is in the shape of a circle with a small segment removed as shown. 3
The circle has centre O and radius 0.6 metres. The length of the straight edge AB is also 0.6 metres.
Find the area of the table-top, leaving your answer in exact form.



① Find $\angle AOB$
→ Let $\angle AOB = \theta$
$\cos\theta = \frac{0.6^2 + 0.6^2 - 0.6^2}{2 \times 0.6^2}$
$\cos\theta = 0.5$
$\theta = \frac{\pi}{3}$ ✓
→ ΔAOB is equilateral
$\therefore \pi \div 3 = \frac{\pi}{3}$
$\therefore \angle AOB = \frac{\pi}{3}$ ✓

② Area = $\text{Area}_{circ} - \text{Area}_{seg}$
$= (\pi \times 0.6^2) - \left(\frac{1}{2} \times 0.6^2 \times \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\right)$
$= \frac{9\pi}{25} - \left(0.18\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)$
$= \frac{9\pi}{25} - \frac{3\pi}{50} + \frac{9\sqrt{3}}{100}$
$= \frac{3\pi}{10} + \frac{9\sqrt{3}}{100} \; m^2$ ③ Exact form

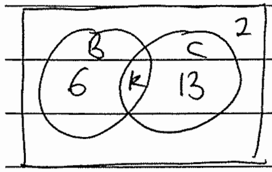
Question Twenty-Three

A small group of people were surveyed to determine whether they were part of the local basketball or curling team. 3

- 6 people played basketball only.
- 13 people played curling only.
- 3 people played neither.
- k people played both.

Let B be the event that a person plays basketball and let C be the event that a person plays curling.
Determine the value of k such that the events B and C are independent.

[**Curling**: a game played on ice, especially in Scotland and Canada, in which large round flat stones are slid across the surface towards a mark. Members of a team use brooms to sweep the surface of the ice in the path of the stone to control its speed and direction.]



Independent:
$P(B \cap C) = P(B) \cdot P(C)$

$$\frac{k}{k+22} = \frac{6+k}{k+22} \times \frac{13+k}{k+22}$$

$$k(k+22) = (6+k)(13+k)$$

$$k^2+22k = 78+19k+k^2$$

$$3k = 78$$

$$k = 26$$

Question Twenty-Four

A particle moves on a straight line. When it is x metres from the origin, its velocity is given by $v = 4 - e^{\frac{-t}{2}}$, the time t being measured in seconds.

a) Find its acceleration at time t. 1

$$a = \frac{e^{-\frac{t}{2}}}{2}$$

b) Initially the particle was at the origin. Find its displacement from the origin after 4 seconds. Leave your answer in metres, correct to 2 decimal places. 3

$$x = \int 4 - e^{-t/2}\, dt$$

$$x = 4t + 2e^{-t/2} + C$$

when $t=0$ $x=0$

$$0 = 0 + 2 + C$$

$$C = -2$$

$$x = 4t + 2e^{-t/2} - 2$$

when $t = 4$

$$x = 4(4) + 2e^{-2} - 2$$

$$x = 14 + \frac{2}{e^2}$$

$= 14.27$m to the right of the origin.

Question Twenty-Five

Sketch the graph of $y = x^3 - 6x^2 + 9x - 5$ in the space provided showing all: **5**

- turning points
- points of inflexion
- y-intercept

There is no need to find the x-intercept(s).

when $x = 0$

$y = -5$

$\therefore$ y-int : $(0, -5)$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

when $\frac{dy}{dx} = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, x = 1$

$\frac{d^2y}{dx^2} = 6x - 12$

when $x = 3$

$\frac{d^2y}{dx^2} = 18 - 12$

> 0

$\therefore$ min.

when $x = 1$

$\frac{d^2y}{dx^2} = 6 - 12$

$< 0 \quad \therefore$ max

$\therefore x = 3, \; y = 3^3 - 6(3)^2 + 9(3) - 5$

$= -5$

$\therefore (3, -5)$ - minimum

$x = 1, \; y = 1^3 - 6(1)^2 + 9(1) - 5$

$= -1$

$\therefore (1, -1)$ max.

when $\frac{d^2y}{dx^2} = 0$

$6x - 12 = 0$

$x = 2$

x	1	2	3
$\frac{d^2y}{dx^2}$	$\cap$	$-$	$\cup$

$\therefore$ change in concavity

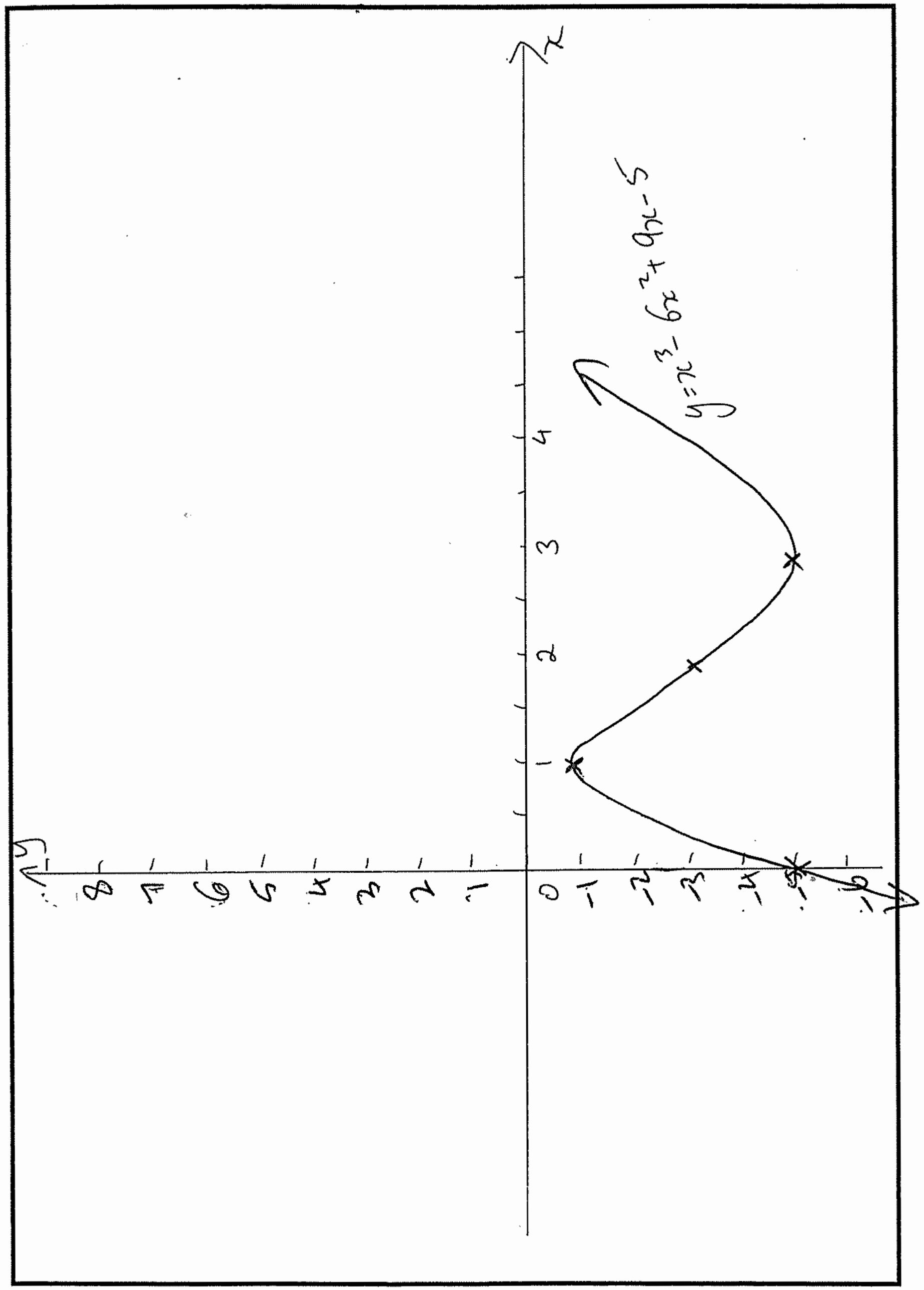
$\therefore$ point of inflexion

when $x = 2$

$y = 2^3 - 6(2)^2 + 9(2) - 5$

$= -3$

$\therefore (2, -3)$ point of inflexion.



Question Twenty-Six

A probability density function f for a random variable X is given by:

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{9+2x}}, & 0 \le x \le 8 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

a) Show that $k = \frac{1}{2}$. 2

$\int_0^8 \frac{k}{\sqrt{9+2x}}\,dx = 1$

$\frac{k}{2}\int_0^8 \frac{2}{\sqrt{9+2x}}\,dx = 1$

$\frac{k}{2}\left[\sqrt{9+2x}\right]_0^8 = 1$

$\frac{k}{2}\left[\left[\sqrt{9+2(8)}\right] - \left[\sqrt{9+2(0)}\right]\right] = 1$

$\frac{k}{2}\left(\sqrt{25} - \sqrt{9}\right) = 1$

$\frac{k}{2}(5-3) = 1$

$\frac{k}{2} \cdot 2 = 1$

$k = \frac{1}{2}$

b) Find the cumulative distribution function $F(x)$. 2

$\int_0^x \frac{1}{2\sqrt{9+2x}}\,dx = \frac{1}{2}\left[\sqrt{9+2x}\right]_0^x$

$= \frac{1}{2}\left[\sqrt{9+2x} - \sqrt{9-2(0)}\right]$

$= \frac{1}{2}\left(\sqrt{9+2x} - 3\right)$

$= \frac{\sqrt{9+2x} - 3}{2}$

c) Find the median. 2

let $F(x) = 0.5$

$\frac{\sqrt{9+2x} - 3}{2} = 0.5$

$\sqrt{9+2x} - 3 = 1$

$\sqrt{9+2x} = 4$

$9+2x = 16$

$2x = 7$

$x = \frac{7}{2}$

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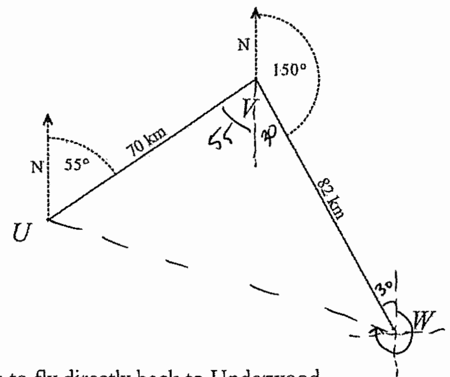
Question Twenty-Seven

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vanna Beach.
It then flies 82 km on a bearing of 150° to Weston.
The diagram below illustrates the journey.



a) It then plans to fly directly back to Underwood. Calculate, to the nearest km, the distance that it will fly. 2

$\angle UVW = 55 + 30$

$= 85$

$UW^2 = 70^2 + 82^2 - 2(70)(82)\cos 85$

$UW^2 = 10623.45207...$

$UW = 103 \text{ km}$

b) Determine the bearing from Weston on which it should fly to return to Underwood. Give your answer to the nearest degree. 2

$\frac{\sin(\angle VWU)}{70} = \frac{\sin 85}{103}$

$\sin\angle VWU = \frac{70 \sin 85}{103}$

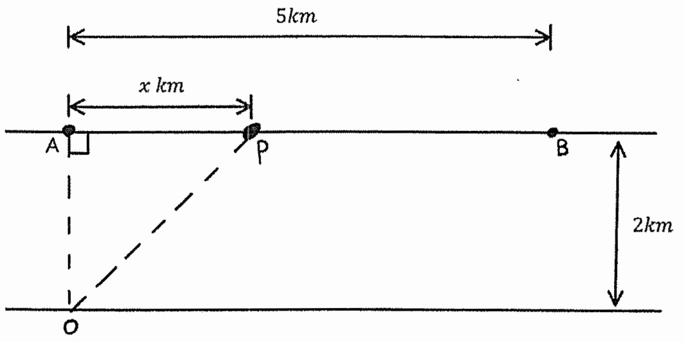
$\angle VWU = 42°36'41.91''...$

$\text{Bearing} = 360 - 30 - 42°36'41.91''...$

$= 287°T$

Question Twenty-Eight

The diagram shows a straight section of a river, two kilometres wide. Gonzo is at a point O on one bank and he wishes to reach a point B on the opposite bank. The point A is directly opposite O and the distance from A to B is 5 kilometres.
Gonzo can row at a speed of 8 km/h, and jog at 12 km/h. He intends to row in a straight line to a point P on the opposite bank and then jog directly from P to B.
Let the distance AP be x km.



a) Show that the time T, in hours, that Gonzo takes to reach B is given by $T = \frac{\sqrt{x^2+4}}{8} + \frac{5-x}{12}$. **2**

D / S | T

$OP^2 = 2^2 + x^2$

$OP = \sqrt{4+x^2}$ | $PB = 5-x$

$T = \frac{D}{S}$ | $T_{PB} = \frac{5-x}{12}$

$\therefore T_{OP} = \frac{\sqrt{4+x^2}}{8}$ | $\therefore$ Total time $= \frac{\sqrt{4+x^2}}{8} + \frac{5-x}{12}$

b) Find the distance AP for which Gonzo's journey is minimised. **4**

$T = \frac{1}{8}\sqrt{4+x^2} + \frac{5}{12} - \frac{x}{12}$

$\frac{dT}{dx} = \frac{1}{2}\cdot\frac{1}{8}\cdot 2x\cdot(4+x^2)^{-1/2} - \frac{1}{12}$

$= \frac{x}{8\sqrt{4+x^2}} - \frac{1}{12}$

when $\frac{dT}{dx} = 0$

$\frac{x}{8\sqrt{4+x^2}} - \frac{1}{12} = 0$

$\frac{x}{8\sqrt{4+x^2}} = \frac{1}{12}$

$12x = 8\sqrt{4+x^2}$

$\frac{3x}{2} = \sqrt{4+x^2}$

$\frac{9x^2}{4} = 4+x^2$

$9x^2 - 4x^2 = 16$

$5x^2 = 16$

$x^2 = \frac{16}{5}$

$x = \pm\frac{4}{\sqrt{5}}$

x	-2	$-\frac{4}{\sqrt{5}}$	0	$\frac{4}{\sqrt{5}}$	2
$\frac{dT}{dx}$	\	—	\	—	/

$\therefore x = \frac{4}{\sqrt{5}}$ is a minimum.

$\therefore$ Journey is minimised when $x = \frac{4}{\sqrt{5}}$ m

Question Twenty-Nine

Data was collected from 6 people detailing the number of black cats they have seen per year and the average number of car accidents they are involved in per year.

Number of black cats seen in a year (x)	4	6	8	13	17	20
Number of car accidents per year (y)	1	4	2	5	7	9

a) Calculate Pearson's correlation coefficient correct to 2 d.p. 1

$r = 0.9449713...$

$r = 0.94$

b) Write the equation of the least squares regression line, writing your values to 2 d.p. 1

$A = -0.38688...$ | $B = 0.445901..$ | $y = 0.45x - 0.39$

$A = -0.39$ | $B = 0.45$ | ✓

c) If a person was in 6 car accidents in a year, how many black cats would you expect them to have seen. Answer to the nearest cat. 1

when $y = 6$ | $0.45x = 6.39$ | $\therefore$ 14 cats

$6 = 0.45x - 0.39$ | $x = 14.2$ | expected to be seen

d) Antonia looked at this data and made the statement "Black cats are unlucky, therefore the more that a person has seen, the more car accidents they have been in." 1
Based on the data and your knowledge of statistics, justify whether you agree or disagree with this statement.

Just disagree, No Mark. →

Disagree. Although the correlation coefficient implies a strong positive correlation, there is no link between cats and car accidents.
Correlation does not imply causation!

Question Thirty

Data collected for a Speedrun of the video game Legend of Zelda: Ocarina of Time resulted in a mean of 4 hours 40 minutes 20 seconds, with a standard deviation of 12 minutes 27 seconds. Elliot saw that there was a competition with prizes on offer for any entry that can complete the Speedrun faster than 93% of people. 3

An extract from a probability table for the standard normal distribution is shown below.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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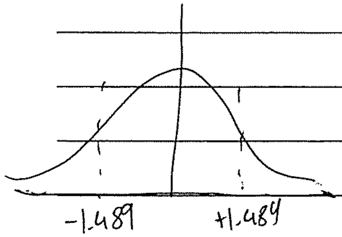
Elliot completed the Speedrun in 4 hours 21 minutes and 47 seconds.
Did he qualify for a prize?
Justify your answer through your knowledge of the normal distribution and z-scores.

$Z = \dfrac{4°21°47° - 4°40°20°}{0°12°27°}$

$= -1.4899...$

$P(z < 1.489) = 6.94\%$

(93.06% "below" him)



$\therefore$ Yes, he recieves a prize as he is faster than 93% of applicants

Question Thirty-One

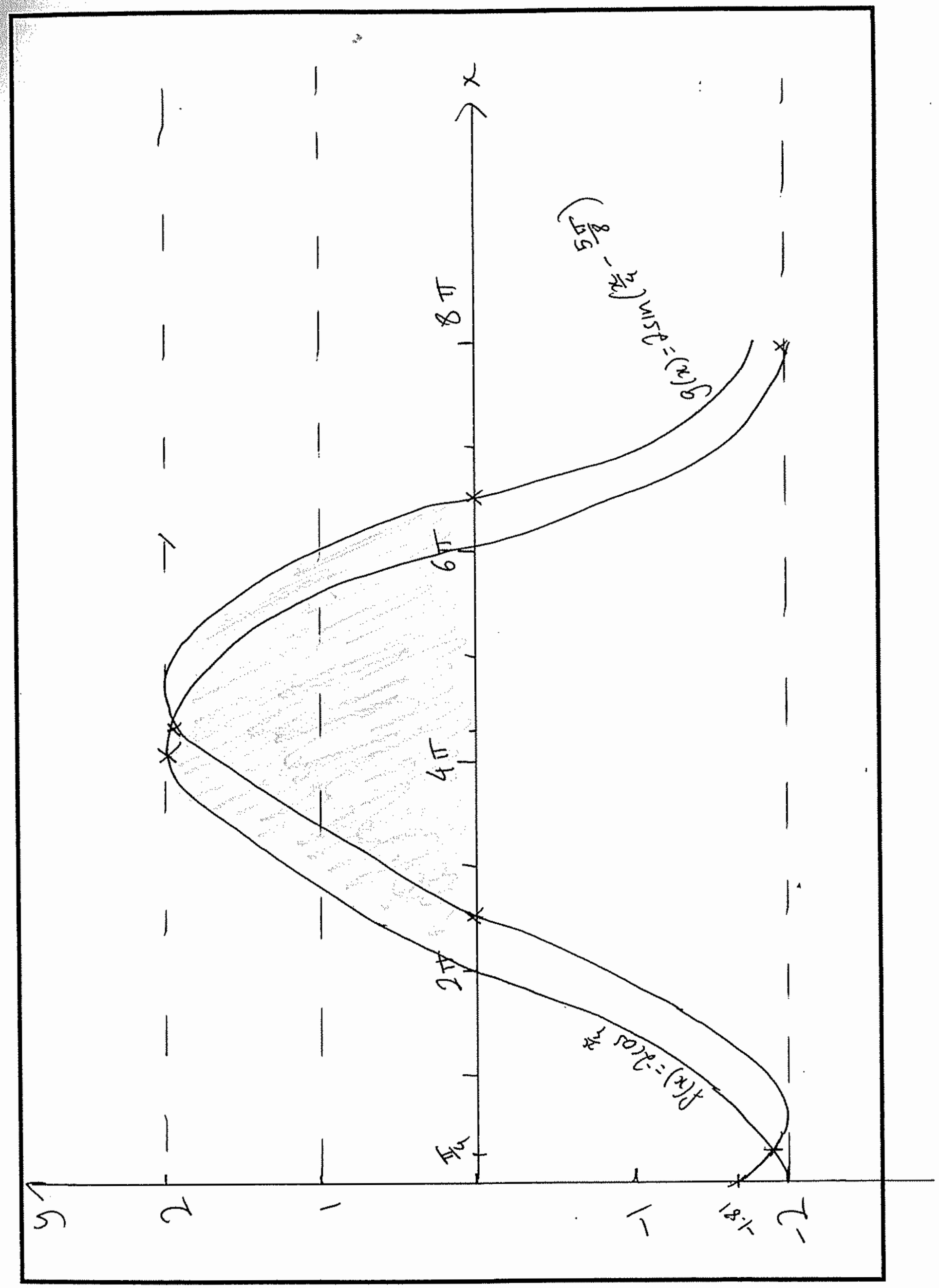
The planet Tatooine has Twin Suns that it orbits around.
The rising and setting of these suns can be modelled with the functions:

Sun 1	$f(x) = -2\cos\frac{x}{4}$
Sun 2	$g(x) = 2\sin(\frac{x}{4} - \frac{5\pi}{8})$

a) Neatly sketch the two equations in the space provided, between $0 \leq x \leq 8\pi$. **3**
Note: The two graphs intersect when $x = \frac{\pi}{4}$ and when $x = \frac{17\pi}{4}$ in the given domain. DO NOT PROVE THIS.

$f(x) = -2\cos\frac{x}{4}$
amp $= 2$
period $= \frac{2\pi}{1/4}$
$= 8\pi$
when $f(x) = 0$
$-2\cos\frac{x}{4} = 0$
$\frac{x}{4} = \frac{\pi}{2}, \frac{3\pi}{2}$
$x = 2\pi, 6\pi$
when $x = 0$
$f(x) = -2$

$g(x) = 2\sin\left(\frac{x}{4} - \frac{5\pi}{8}\right)$
$g(x) = 2\sin\left(\frac{1}{4}\left(x - \frac{5\pi}{2}\right)\right)$
period $= 2\pi/1/4$
$= 8p$
amp $= 2$
phase $= \frac{5\pi}{2}$
when $g(x) = 0$
$2\sin\left(\frac{x}{4} - \frac{5\pi}{8}\right) = 0$
$\frac{x}{4} - \frac{5\pi}{8} = 0, \pi, 2\pi$
$\frac{x}{4} = \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{21\pi}{8}$
$x = 2.5\pi, 6.5\pi, 10.5\pi$
when $x = 0$
$g(x) = -1.81$



b) The horizon is represented by the x-axis.
Work cannot be done on the planet when either of the two suns are in the top half of their path across the sky. For how many hours must work stop each day? **3**

Halfway up horizon when $y = 1$

$1 = -2\cos\frac{x}{4}$

$\cos\frac{x}{4} = -\frac{1}{2}$

$\frac{x}{4} = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad 0 \le x \le 8\pi$ $\quad 0 \le \frac{x}{4} \le 2\pi$

$x = \frac{8\pi}{3}, \frac{16\pi}{3}$

$= 480°, 960°$

$= 8h, 16h$

$1 = 2\sin\left(\frac{x}{4} - \frac{5\pi}{8}\right)$

$\sin\left(\frac{x}{4} - \frac{5\pi}{8}\right) = \frac{1}{2}$

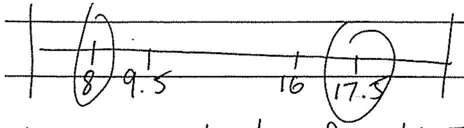
$\frac{x}{4} - \frac{5\pi}{8} = \frac{\pi}{6}, \frac{5\pi}{6}$

$\frac{x}{4} = \frac{19\pi}{24}, \frac{35\pi}{24}$

$x = \frac{19\pi}{6}, \frac{35\pi}{6}$

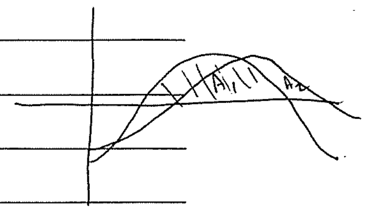
$= 570, 1050$

$= 9.5, 17.5$



~~work must stop for $17.5 - 8 = 9.5$ hours~~ each day.

c) If the area under the curves bounded by the x-axis represents the total surface area of Tatooine experiencing sunlight (in millions of km^2), find the area of the planet that receives daylight for the given cycle. Give your answer to 2 d.p. **4**



$A_1 = \int_{2\pi}^{6\pi} -2\cos\frac{x}{4}\,dx$

$= \left[-8\sin\frac{x}{4}\right]_{2\pi}^{6\pi}$

$= \left[-8\sin\frac{3\pi}{2} + 8\sin\frac{\pi}{2}\right]$

$= 16$

$A_2 = \int_{\frac{17\pi}{4}}^{\frac{13\pi}{2}} 2\sin\left(\frac{x}{4} - \frac{5\pi}{8}\right)dx - \int_{\frac{17\pi}{4}}^{6\pi} -2\cos\left(\frac{x}{4}\right)dx$

$= \left[-8\cos\left(\frac{x}{4} - \frac{5\pi}{8}\right)\right]_{\frac{17\pi}{4}}^{\frac{13\pi}{2}} - \left[-8\sin\frac{x}{4}\right]_{\frac{17\pi}{4}}^{6\pi}$

$= \left[\left(-8\cos\left(\frac{13\pi}{8} - \frac{5\pi}{8}\right)\right) - \left(-8\cos\left(\frac{17\pi}{16} - \frac{5\pi}{8}\right)\right)\right] -$

$\left[-8\sin\frac{6\pi}{4} + 8\sin\frac{17\pi}{16}\right]$

$= 9.560722576\ldots - 6.439277424\ldots$

$= 3.121445152\ldots$

$\therefore$ Total Area $= 16 + 3.121445152\ldots$

$= 19.12$ (million) km^2

-End of Examination-